國立中正大學
115 學年度碩士班招生考試
試 題
[第2節]

科目名稱	數學
系所組別	資訊工程學系

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

1 (5 pt) Prove or disprove that A is row equivalent to I.

$$A = \begin{bmatrix} \pi & 0 & 0 & 0 \\ 13 & 2 & 0 & 0 \\ 5 & 105 & 1 & 0 \\ e & 3 & 11 & 7.7 \end{bmatrix}$$

2 (5 pt) Given two non-overlapped spheres P and Q, determine whether there exists an invertible matrix transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ such that the images of P and Q under T are overlapped. Briefly explain your answer.

3 (10 pt) Determine whether the polynomial

$$p = 1 + x + x^2 + x^3$$

is in the span of $p_1 = 8 - 2x + x^2 - 4x^3$, $p_2 = -3 + 9x + 11x^2 + 6x^3$, and $p_3 = 13 - x + 2x^2 + 4x^3$.

4 (5 pt) Is there a 3×3 symmetric matrix with eigenvalues $\lambda_1 = -1$, $\lambda_2 = 3$, $\lambda_3 = 7$, and corresponding eigenvectors $v_1 = [0 \quad 1 \quad -1]$, $v_2 = [2 \quad 0 \quad 0]$, $v_3 = [-3 \quad 2 \quad -2]$? Explain your reasoning.

5 Suppose we know that $Ax = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$ and the general solution of x is $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

(1) (1 pt) What is the dimension of the row space of A?

(2) (4 pt) What is A?

6 For each of the following, provide or disprove that it is an inner product on the given real vector space.

(1) (3 pt) $\langle (a,b),(c,d) \rangle = ac - bd$ in $\mathbb{R}^2$.

(2) (3 pt) $\langle A,B \rangle = \mathrm{tr}(A+B)$ on the vector space of 2×2 real matrices.

(3) (4 pt) $\langle f,g \rangle = \int_0^1 f' g \mathrm{d}x$ on the vector space of polynomials. Note that f' is the first derivative of f.

7 Consider the following matrix.

$$A = \begin{bmatrix} 2 & -2 & 1 \\ -4 & -9 & -8 \end{bmatrix}$$

Suppose we know that $A^T A$ can be orthogonally diagonalized as QDQ^T, in which

$$D = \begin{bmatrix} 144 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } Q = \begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & -2/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \end{bmatrix}.$$

(1) (8 pt) Find the reduced singular value decomposition of A.

(2) (2 pt) Compute the rank 1 approximation of A.

8 Use the rules of inference to construct a valid argument showing that the conclusion "Someone who passed the first exam has not read the book." follows from the premises:

"A student in this class has not read the book."

"Everyone in this class passed the first exam."

Let $C(x)$ denote "x is in this class", $B(x)$ denote " x has read the book", and $P(x)$ denote "x passed the first exam".

(1) (5 pt) Translate the premises and conclusion into symbolic form.

(2) (10 pt) Construct a valid argument step by step. You need to provide the rules of inference or reason in each step.

9.

(1) (5 pt) What is the number of possible ways to parenthesize the product of 4 numbers $x_0 \cdot x_1 \cdot x_2 \cdot x_3$? List them all.

(2) (10 pt) Find a recurrence relation for C_n, the number of ways to parenthesize the product of $n + 1$ numbers, $x_0 \cdot x_1 \cdot x_2 \cdot \cdots \cdot x_n$, to specify the order of multiplication. You also need to provide the initial conditions for the recurrence relation.

10. (10 pt) Show that Every infinite set contains a countably infinite subset.

11. (10 pt) Let G be a graph with adjacency matrix A with respect to the ordering $v_1, \ldots, v_n$ of vertices (with directed or undirected edges, multiple edges and loops allowed). Prove by mathematical induction that the number of different paths of length r from v_i to v_j, where $r > 0$ is a positive integer, equals the (i,j)th entry of A^r.