

國立中正大學

113 學年度碩士班招生考試

試題

[第 1 節]

科目名稱	數學
系所組別	資訊工程學系-甲組、乙組

—作答注意事項—

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。
2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。
3. 入場後於考試開始 40 分鐘內不得離場。
4. 全部答題均須在試卷（答案卷）作答區內完成。
5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。
6. 試題須隨試卷繳還。

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1. (10 points) Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be nonzero vectors in R^3 with the same initial point and $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = -4$. Which of the following statements are correct? Note that there may be *multiple* answers to this question.

- (a) $\mathbf{v} \cdot (\mathbf{u} \times \mathbf{w}) = -4$
- (b) $\mathbf{v} \cdot (\mathbf{w} \times \mathbf{w}) = 0$
- (c) $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ lies in the plane determined by \mathbf{v} and \mathbf{w} .
- (d) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ lies in the plane determined by \mathbf{u} and \mathbf{v} .
- (e) The vectors $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$ and $\mathbf{u} \times (\mathbf{v} \times \mathbf{w})$ are the same.

2. (10 points) Which of the following are subspaces of R^3 ? Note that there may be *multiple* answers to this question.

- (a) All vectors of the form $(a, 0, 0)$.
- (b) All vectors of the form $(a, 1, 0)$.
- (c) All vectors of the form (a, b, c) where $b = a + c$.
- (d) All vectors of the form (a, b, c) where $c = a - b$.
- (e) All vectors of the form $(a, -a, 0)$.

3. (10 points) Let $\mathbf{u} = \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \\ 9 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 11 \\ 13 \\ 15 \\ 17 \\ 19 \end{bmatrix}$. Which of the following is the rank of $\mathbf{u}\mathbf{v}^T$?

- (a) 1
- (b) 2
- (c) 3
- (d) 4
- (e) 5

4. (10 points) Which of the following are the eigenvalues of \mathbf{A}^7 ? Note that there may be *multiple* answers to this question.

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 8 & 7 & 0.5 & 0 \\ -1 & 9 & 6 & 0 \end{bmatrix}$$

- (a) 1
- (b) -1
- (c) 2
- (d) 64
- (e) 128

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科目名稱：數學

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系所組別：資訊工程學系-甲組、乙組

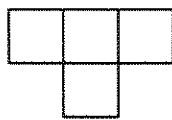
5. (10 points) Sketch the unit circle in R^2 using the following inner product.

$$\mathbf{u} \cdot \mathbf{v} = \frac{1}{4}u_1v_1 + \frac{1}{16}u_2v_2$$

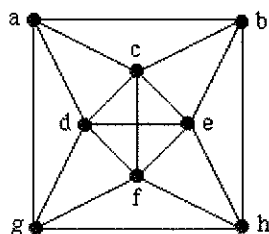
6. (10 points) Assume that the universe for x is all people and the universe for y is the set of all movies. Write the statement using the following predicates and any needed quantifiers:

$S(x, y)$: x saw y $L(x, y)$: x liked y $A(y)$: y won an award $C(y)$: y is a comedy.

- a) (2 points) No comedy won an award.
 - b) (2 points) Lois saw Casablanca, but didn't like it.
 - c) (2 points) Some people have seen every comedy.
 - d) (2 points) No one liked every movie he has seen.
 - e) (2 points) Ben has never seen a movie that won an award.
7. (10 points) A *T-omino* is a tile pictured as follows. Prove that every $2^n \times 2^n$ ($n > 1$) chessboard can be tiled with T-ominoes.



8. (10 points) Determine whether the following graph is planar or not. Provide proofs or reasons.



9. (20 points) Fill the blank. Each blank is 2 point.
- a) If T is a tree with 999 vertices, then T has _____ edges.
 - b) There are _____ non-isomorphic rooted trees with four vertices.
 - c) Write $3n - (k + 5)$ in prefix notation: _____.
 - d) A cycle graph C_7 has _____ spanning trees.
 - e) If each edge of n -dimensional hypercube Q_4 has weight 1, then the cost of any spanning tree of minimum cost is _____.
 - f) If T is a full binary tree with 101 vertices, its minimum height is _____.
 - g) Every full binary tree with 50 leaves has _____ vertices.
 - h) The incidence matrix for the wheel graph W_n has _____ rows and _____ columns.
 - i) List all positive integers n such that the complete graph K_n has an Euler circuit. _____
 - j) List all positive integers m and n such that the complete bipartite graph $K_{m,n}$ has a Hamilton path but no Hamilton circuit. _____