國立中正大學
114 學年度碩士班招生考試
試 題

［第 2 節］

科目名稱	數學
系所組別	資訊工程學系-甲組、乙組

－作答注意事項－

※作答前請先核對「試題」、「試卷」與「准考證」之系所組別、科目名稱是否相符。

1. 預備鈴響時即可入場，但至考試開始鈴響前，不得翻閱試題，並不得書寫、畫記、作答。

2. 考試開始鈴響時，即可開始作答；考試結束鈴響畢，應即停止作答。

3. 入場後於考試開始 40 分鐘內不得離場。

4. 全部答題均須在試卷（答案卷）作答區內完成。

5. 試卷作答限用藍色或黑色筆（含鉛筆）書寫。

6. 試題須隨試卷繳還。

1. (10%) Find the standard matrix A for the linear transformation $T: R^2 \rightarrow R^2$ for which

    $$T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\-2\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}2\\3\end{bmatrix}\right) = \begin{bmatrix}-2\\5\end{bmatrix}.$$

2. (10%) Let V be the subspace of P_4 spanned by $S = \{v_1, v_2, v_3, v_4\}$, where $v_1 = t^4 + t^2 + 2t + 1$, $v_2 = t^4 + t^2 + 2t + 2$, $v_3 = 2t^4 + t^3 + t + 2$, and $v_4 = t^4 + t^3 - t^2 - t$. Find a basis for V.

3. (15%) Find bases for the row space and the column space of the matrix:

    $$A = \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ -2 & 5 & -7 & 0 & -6 \\ -1 & 3 & -2 & 1 & -3 \\ -3 & 8 & -9 & 1 & -9 \end{bmatrix}$$

4. (15%) Let $A = \begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix}$.

    Confirm that P diagonalizes A, and then compute A^{11}.

5. (16%) Determine the truth value of each of the following statements if the universe of discourse of each variable consists of all real numbers.
    a) $\forall x \exists y (x^2 = y)$
    b) $\forall x \exists y (x = y^2)$
    c) $\forall x (x^2 \neq x)$
    d) $\forall x (|x| > 0)$
    e) $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$
    f) $\forall x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$
    g) $\forall x \forall y (x + 2y = 3 \wedge 2x + 4y = 6)$
    h) $\exists x \exists y (x + 2y = 2 \wedge 2x + 3y = 3)$

6. (9%) How many numbers must be selected from the first 12 positive integers to guarantee that at least three pairs of these numbers add up to 15? Explain how to do?
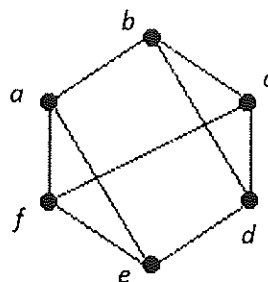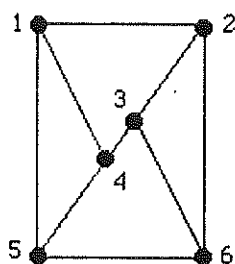
7. (13%) Solve the recurrence relation $a_n^2 - 2a_{n-1}^2 = 1$ for $n \geq 1$, where $a_0 = 1$.

8. a) (6%) Determine whether the following two graphs are isomorphic? If YES, provide the corresponding vertex pair. Otherwise, justify your answer with detailed explanations.

b) (6%) In a complete graph K_5, find the number of paths of length 6 between every pair of vertices.